

# Extended tachyon field, Chaplygin gas and solvable $k$ -essence cosmologies

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## Abstract

We investigate a flat Friedmann-Robertson-Walker spacetime filled with  $k$ -essence and find the set of functions  $F$  which generate three different families of extended tachyon fields and Chaplygin gases. They lead to accelerated and superaccelerated expanding scenarios.

For any function  $F$ , we find the first integral of the  $k$ -field equation when the  $k$ -field is driven by an inverse square potential or by a constant one. In the former, we obtain the general solution of the coupled Einstein- $k$ -field equations for a linear function  $F$ . This model shares the same kinematics of the background geometry with the ordinary scalar field one driven by an exponential potential. However, they are dynamically different. For a constant potential, we introduce a  $k$ -field model that exhibits a transition from a power-law phase to a de Sitter stage, inducing a modified Chaplygin gas.

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## I. INTRODUCTION

Cosmological inflation has become an integral part of the standard model of the universe and perhaps the only known mechanism which can dynamically solve the flatness and the horizon problem of the universe [1]. It has gained certain support from the recent observations of the cosmic microwave background anisotropies [2]-[4]. Although particle physics, in particular M/String theory, provides several very weakly coupled scalar fields which are natural inflaton candidates, there exists no clearly preferred inflationary model. A link between string theory and inflation was investigated in Ref. [5] where the authors introduce the  $k$ -inflation and show that the  $k$ -field may drive an inflationary evolution starting from rather generic initial conditions. Also, the process of rolling tachyon field has been extensively used to implement power-law accelerated expansion scenarios [6]-[11]. In Ref. [12] it was pointed out that rolling tachyons can contribute a mass density to the universe that resembles classical dust. This has brought a new understanding of the role of the tachyon in string theory. The tachyonic matter, then, might provide an explanation for inflation at the early epochs and could contribute to some new form of cosmological dark energy [13] at late times [14]. These facts suggest to enlarge the theoretical background exploring some new possibilities, in this sense, we will introduce an extended tachyon field (ETF) which supply accelerated and superaccelerated expanding scenarios.

In Ref. [15] it was argued that the coincidence problem may be solved assuming a universe filled with a viscous fluid and dark energy modelled with a tachyonic field or a Chaplygin gas. The conjecture that dark matter and dark energy can be unified by using a generalized Chaplygin gas obeying an exotic equation of state has been investigated in several works in view of the cosmological observations [16]-[22]. The present situation is somewhat controversial, with some tests indicating good agreement between observational data and the theoretical predictions of the model and others ruling out the model as an actual possibility of description for our Universe [23]. There were found some differences with the observed CMB and mass power spectrum data. Due to these discrepancies, it may be useful to consider other candidates for dark energy, as for instance, the extended Chaplygin gas (ECG) generated from the ETF. Another interesting possibility appears selecting a  $k$ -field model whose equation of state is like that of two fluids, one obeying a baryotropic equation of state with constant baryotropic index  $\gamma$ , and the other is an ECG. This "modified Chaplygin

gas” interpolates between a power-law phase and a de Sitter phase.

From theoretical and experimental points of view it is important to find an exact shape of the potential, for instance, tachyonic inflation has been studied using phenomenological potentials that have not been derived from string theory and can be related to the so called “k-Inflation” [5], [7, 8]. Such  $k$ -fields, described by a non-standard kinetic term, has been one of the recent suggested candidates to play the role of same unknown component of the universe. One of the purpose of introducing  $k$ -essence is to provide a dynamical explanation of cosmological observations which does not require the fine-tuning of initial conditions [24]-[26]. In this sense, it may be considered as an alternative to quintessence, which requires a scalar field that slowly rolls down a potential to depict the observed acceleration of the present universe. Also, it was argued that in certain dynamical regimes the  $k$ -essence is equivalent to quintessence and it may prove difficult to distinguish between the two fields. In the light of these results, it seems clear that it is worth to search the links between scalar field and  $k$ -essence cosmologies, in especial, whether they are kinematically or dynamically equivalent.

In section II, we introduce the ETF and find the power-law solution generated by an inverse square potential. At the end, we define the ECG and show the general behaviour of the background geometry. In section III, we find the first integral of the  $k$ -field equation for an inverse square potential and the general solution of coupled Einstein- $k$ -field equations for a linear function  $F$ . For power-law solutions we demonstrate that the linear  $k$ -field model, with constant function  $F$ , is isomorphic to the model with divergent sound speed. Finally, we introduce the modified Chaplygin gas. In section IV, we show the kinematical equivalence of the  $k$ -field driven by a tachyonic potential and a linear function  $F$ , with the scalar field driven by an exponential potential. The conclusions are stated in section V.

## II. EXTEND TACHYON FIELD AND CHAPLYGIN GAS

The spatially flat homogeneous and isotropic space-time is described by the Friedmann-Robertson-Walker (FRW) line element

$$ds^2 = -dt^2 + a^2(t) \left[ dx_1^2 + dx_2^2 + dx_3^2 \right], \quad (1)$$

where  $a(t)$  is the scale factor. This metric allows a particular calculational simplicity, on

account of both the high degree of symmetry and the single metric degree of freedom. We assume that  $8\pi G = 1$ .

Let us present the Lagrangian of the  $k$ -essence field

$$\mathcal{L} = -V(\phi) F(x), \quad x = g^{ik} \phi_i \phi_k, \quad (2)$$

where  $V(\phi)$  is a positive defined potential,  $F(x)$  is an arbitrary function of  $x$ ,  $\phi$  is the  $k$ -field and  $\phi_i = \partial V(\phi)/\partial x^i$ . Associating the energy-momentum tensor of the  $k$ -field,

$$T_{ik} = V(\phi) [2F_x \phi_i \phi_k - g_{ik} F], \quad F_x = \frac{dF}{dx}, \quad (3)$$

with the energy-momentum tensor of a perfect fluid, we compute the energy density  $\rho$ , the pressure  $p$  and the baryotropic index  $\gamma = 1 + p/\rho$  of this equivalent fluid

$$\rho = V(\phi)[F - 2xF_x], \quad p = \mathcal{L} = -V(\phi)F, \quad (4)$$

$$\gamma = -\frac{2\dot{H}}{3H^2} = -\frac{2xF_x}{F - 2xF_x}. \quad (5)$$

From Eq. (4), the Einstein field equations are

$$3H^2 = V[F - 2xF_x], \quad (6)$$

$$\dot{H} = V_x F_x, \quad (7)$$

and the conservation equation reads

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (8)$$

where  $H = \dot{a}/a$  is the expansion rate. Substituting the Eq. (4) into the conservation equation (8), we find the field equation for the  $k$ -field

$$[F_x + 2xF_{xx}]\ddot{\phi} + 3HF_x\dot{\phi} + \frac{V'}{2V}[F - 2xF_x] = 0. \quad (9)$$

The stability of the  $k$ -essence with respect to small wavelength perturbations requires that the effective sound speed

$$c_s^2 = \frac{p_x}{\rho_x} = \frac{F_x}{F_x + 2xF_{xx}}, \quad (10)$$

be positive. However, in Ref. [27] it was shown that a positive sound speed is not a sufficient condition for the theory to be stable. For the tachyon field, which is obtained from the  $k$ -field selecting  $F = (1 - \dot{T}^2)^{1/2}$ , with  $\dot{T}^2 = -x = \dot{\phi}^2$  the sound speed is  $c_s^2 = 1 - \gamma > 0$ , where  $\gamma = \dot{T}^2$ . Let us look for the set of functions  $F$ , such that,  $c_s^2$  is proportional to the sound speed of the tachyon field. We express this proportionality as

$$c_s^2 = \frac{1 - \gamma}{2r - 1}, \quad (11)$$

where  $r$  is a real constant, with  $r < 1/2$  for  $\gamma > 1$  and  $r > 1/2$  for  $\gamma < 1$ . From Eqs. (5), (10) and (11), we obtain a differential equation for the function  $F$

$$(1 - r)FF_x + (2r - 1)xF_x^2 + xFF_{xx} = 0. \quad (12)$$

Integrating for  $r = 0$ , we obtain the set of functions  $F_0(x) = x^{\gamma_0/2(\gamma_0-1)}$ , with  $\gamma_0$  constant, investigated in [28]; while for  $r = 1$  we find three types of solution  $F_1 = (1 - \dot{T}_1^2)^{1/2}$ ,  $F_1 = (1 + \dot{T}_1^2)^{1/2}$ , and  $F_1 = -(\dot{T}_1^2 - 1)^{1/2}$ . The first function corresponds to the ordinary tachyon field  $T = T_1$ . Its energy density  $\rho_1 = VF_1^{-1}$  and pressure  $p_1 = -VF_1$  leads to the relation  $p_1 = (\gamma_1 - 1)\rho_1$ , where  $\gamma_1 = \dot{T}_1^2$ . The other two functions generate two "new tachyon field" which will be investigated below. For the remaining values of  $r$  the general solution of Eq. (12) is given by  $F_r^{2r} = c_1(-x)^r + c_2$ , where  $c_1$  and  $c_2$  are arbitrary integration constants, the baryotropic index (5) is  $\gamma_r = -c_1(-x)^r/c_2$  and we have  $F_r^{2r} = c_2(1 - \gamma_r)$ . From Eqs. (4) and (5), the energy density of the  $k$ -field can be rewritten as  $\rho = VF/(1 - \gamma)$ . So, without loss of generality, we can split the solutions as follow: for  $\gamma_r < 1$  and  $r > 1/2$ , it is necessary that  $F_r > 0$  and  $c_2 > 0$ , with two options,  $0 < \gamma_r < 1$  and  $c_1 < 0$  or  $\gamma_r < 0$  and  $c_1 > 0$ ; for  $\gamma_r > 1$  and  $r < 1/2$ , we have  $F_r < 0$ ,  $c_2$  and  $c_1 > 0$ . Finally

$$F_r = (1 - \gamma_r)^{1/2r}, \quad \gamma_r = \pm \dot{T}_r^{2r} < 1, \quad (13)$$

$$F_r = -(\gamma_r - 1)^{1/2r}, \quad \gamma_r = \dot{T}_r^{2r} > 1, \quad (14)$$

where  $\gamma_r$  is the extended baryotropic index and the  $T_r$ -field linked with  $F_r$  will be called ETF. Inserting Eqs. (13) and (14) into (4), we get the energy density  $\rho_r$  and the pressure  $p_r$  of each ETF

$$\rho_r = V(1 - \dot{T}_r^{2r})^{(1-2r)/2r}, \quad p_r = -V(1 - \dot{T}_r^{2r})^{1/2r}, \quad 0 < \gamma_r < 1, \quad (15)$$

$$\rho_r = V(1 + \dot{T}_r^{2r})^{(1-2r)/2r}, \quad p_r = -V(1 + \dot{T}_r^{2r})^{1/2r}, \quad \gamma_r < 0, \quad (16)$$

$$\rho_r = V(\dot{T}_r^{2r} - 1)^{(1-2r)/2r}, \quad p_r = V(\dot{T}_r^{2r} - 1)^{1/2r}, \quad 1 < \gamma_r, \quad (17)$$

with

$$p_r = (\gamma_r - 1)\rho_r. \quad (18)$$

The Eq. (15) comprise perfect fluids which generalize the normal tachyon field, which is obtained for  $r = 1$ . The fluids represented by the Eq. (16) with negative pressure and negative baryotropic index describe phantom cosmologies, while those described by the Eq. (17) give rise to nonaccelerated expanding evolutions. In the limit of large  $r$ , the fluids (15) and (16) satisfy the equation of state  $p = -\rho = -V$  acting like a variable cosmological constant depending on the  $k$ -field. Besides, the fluids (17) fulfill the equation of state  $p = \rho = V$  behaving like stiff matter in the same limit. These exotic fluids satisfy the relations

$$p_r = -\frac{V^{2r/(2r-1)}}{\rho_r^{1/(2r-1)}}, \quad \gamma_r < 1, \quad (19)$$

$$p_r = \frac{V^{2r/(2r-1)}}{\rho_r^{1/(2r-1)}}, \quad \gamma_r > 1, \quad (20)$$

which, for a constant potential  $V = V_0$ , become exotic equations of state which extend that of generalized Chaplygin gas.

### A. Power-law expansion for the $T_r$ -field

We begin investigating the atypical behaviour of the  $T_{1/2}$ -field. Its sound speed diverges and the  $k$ -field equation (9) becomes a first order equation. The equation (5) can be inte-

grated and its solution gives a link between the expansion rate and  $T_{1/2}$

$$H = \frac{2}{3T_{1/2}}, \quad (21)$$

where the integration constant has been chosen to set the singularity at  $T_{1/2} = 0$ . For  $r = 1/2$ , the three equations (15), (16) and (17) lead to  $3H^2 = V$ . So, combining them with Eq. (21) we obtain the inverse square potential

$$V_{1/2} = \frac{4}{3T_{1/2}^2}, \quad (22)$$

which is the uniquely allowed potential for  $T_{1/2}$ . As  $T_{1/2}$  is not controlled by the  $k$ -field equation (9), it can be chosen freely and this choice determines, after integrating Eq. (21), the form of the scale factor. Although the potential (22) diverges at  $T_{1/2} = 0$ , it reasonably mimics the behaviour of a typical potential in the condensate of bosonic string theory. One expects the potential to have a unique local maximum at the origin and a unique global minimum away from the origin at which  $V$  vanishes. In the most interesting case the global minimum is taken to lie at infinity. Obviously more complicated potentials may be contemplated but this is the simplest case to begin with.

It will be useful to investigate the existence of accelerated and superaccelerated expanding solutions when the  $T_r$ -field is driven by the potential  $V_r = V_0/T_r^2$ . To do that, let us consider an evolution of the form  $a = t^n$  and a  $T_r$ -field, such that  $T_r \propto t$ . Power law solutions are very important because they can be always obtained from any function  $F$  with an inverse square potential or from polynomial functions  $F = (-x)^N$  with any potential. As these solutions are usual ingredients in the quintessence and  $k$ -essence models they would allow to recognize and compare the differences between the two cosmological models. The complete solution for the ETF is given by

$$T_r = \left(\frac{2}{3n}\right)^{1/2r} t, \quad n = \frac{1}{3} \left[1 + \sqrt{1 + 9\beta^2}\right], \quad 0 < \gamma_r < 1, \quad (23)$$

$$T_r = \left(\frac{2}{-3n}\right)^{1/2r} t, \quad n = \frac{1}{3} \left[1 - \sqrt{1 + 9\beta^2}\right], \quad \gamma_r < 0, \quad (24)$$

$$T_r = \left(\frac{2}{3n}\right)^{1/2r} t, \quad n = \frac{1}{3} \left[1 \pm \sqrt{1 - 9\beta^2}\right], \quad 1 < \gamma_r, \quad (25)$$

where

$$\beta^2 = \left[ \frac{3}{V_0} \left( \frac{2}{3} \right)^{1/r} \right]^{2r/(1-2r)}. \quad (26)$$

The sound speed associated with the above solutions is given by

$$c_s^2 = \frac{1 - 2/3n}{2r - 1}. \quad (27)$$

Assuming a positive sound speed bounded by the light speed  $0 \leq c_s^2 \leq 1$  we can see that  $r > 1 - 1/3n$  is required for  $n > 2/3$  or  $n < 0$  and  $r < 1 - 1/3n$  for  $0 < n < 2/3$ .

An accelerated power-law expanding universe ( $n > 1$ ), can be described by the set of ETF having  $r > 2/3$  and it is represented by the solutions (23). All these models are cinematically and dynamically different because the scale factor and the  $k$ -field are strongly depending of  $r$  as can be seen from Eqs. (23) and (26). In particular, for  $r = 1$  the Eq. (23) reduces to the solution of the tachyon field found in [7, 8]. The solutions (24) with  $n < 0$  describe phantom cosmologies and we have to select ETF with  $r > 1$  to reach this scenario

## B. Extended Chaplygin gas

Recently it was proposed a set of simple cosmological models based on the use of peculiar perfect fluids [29]. In this simple model the universe is filled with the so called Chaplygin gas, which is a perfect fluid characterized by the following equation of state  $p = -A/\rho$ , where  $A$  is a positive constant. It describes a transition from a decelerated cosmological expansion to a cosmic accelerated de Sitter stage. Other possibility is the inhomogeneous Chaplygin gas, which is able to combine the roles of dark energy and dark matter [30], and the generalized Chaplygin gas model discussed in Ref. [31], having two free parameters  $p = -A/\rho^\alpha$  with  $0 < \alpha \leq 1$ .

These cosmological models may give a unified macroscopic phenomenological description of dark energy and dark matter and generalize the usual  $\Lambda$ CDM models. On the other hand, the Chaplygin gas can be considered as the simplest tachyon cosmological models where the tachyon field is a purely kinetic  $k$ -essence model with a constant potential. In the same way, we will show that the generalized Chaplygin gas can be conceived as the simplest ETF model driven by a constant potential. This identification has the advantage of producing a

variety of new Chaplygin gases, some of which leads to superaccelerated scenarios as we will see in this subsection.

Coming back to our Eqs. (19) and (20), we see that in the case of a constant potential  $V(T_r) = V_0$ , with  $r = 1$  and  $0 < \gamma_r < 1$ , the exotic equation of state (19) represents a Chaplygin gas [29]. Such equation may be the consequence of a scalar field with a non-standard kinetic term, e.g., the string theory motivated tachyon field [32, 33]. For  $1 < r$  and  $0 < \gamma_r < 1$ , the equation of state (19) represents a perfect fluid called generalized Chaplygin gas [31]. Finally an ECG will be characterized by the equation of state (19) with  $1/2 < r < 1$ , or by the Eq. (20) with  $r < 1/2$ . Their properties will be investigated below. Using Eqs. (19)-(20) and the relativistic energy conservation equation (8), we obtain the energy density of the ECG:

$$\rho = V_0 \left[ 1 + \left( \frac{a_0^3}{a^3} \right)^{2r/(2r-1)} \right]^{(2r-1)/2r}, \quad 0 < \gamma_r < 1, \quad (28)$$

$$\rho = V_0 \left[ 1 - \left( \frac{a_0^3}{a^3} \right)^{2r/(2r-1)} \right]^{(2r-1)/2r}, \quad \gamma_r < 0, \quad (29)$$

$$\rho = V_0 \left[ -1 + \left( \frac{a_0^3}{a^3} \right)^{2r/(2r-1)} \right]^{(2r-1)/2r}, \quad 1 < \gamma_r. \quad (30)$$

The Eq. (28) with  $r > 1$  gives the energy density of the generalized Chaplygin gas in terms of the scale factor interpolating between a dust dominated phase where the energy density is  $\rho \approx V_0(a_0/a)^3$ , and a de Sitter phase where  $\rho \approx V_0$ . While during the intermediate stage it could be interpreted as a mixture of two-fluids, one of which is the cosmological constant and the other is a perfect fluid with equation of state  $p \propto \rho$ . The additional free parameter  $r$  of the generalized Chaplygin gas can be used to compare it with observational data.

In any other case, Eqs. (28) with  $1/2 < r < 1$ , (29) and (30) represent new perfect fluids which are interesting from the cosmological point of view. The scale factor generated by the source (29) is nonsingular and has a bounce at the minimum  $a = a_0$ . The universe begins from a contracting era and ends in a superaccelerated stage. Such cosmologies may be interpreted as universes filled with baryotropic fluids having a negative constant baryotropic index and violating the weak energy condition  $\rho > 0$ ,  $\rho + p > 0$ . The models will be dubbed phantom cosmologies following the standard terminology. Phantom matter can apparently

be accommodated by current observations [34], and it can be based on the motivation provided by string theory [35]. It looks interesting to admit that the origin of dark energy should be searched within a fundamental theory, as string theory. However, at present there is no consensus whether a universe violating the weak energy condition should generically possess a future singularity or big rip [36].

The source (30) leads to two types of solutions according to the values of the parameter  $r$ . For  $r < 0$  the universe has a finite time span, interpolating between two dust dominated phases and has a maximum at  $a = a_0$ . For  $0 \leq r \leq 1/2$  the universe begins at a singularity with a finite scale factor,  $a = a_0$ , and ends in a dust dominated phase. From the cosmological point of view these solutions are not relevant because they do not describe the present observed accelerated expansion stage.

### III. SOLVABLE K-ESSENCE COSMOLOGIES

In this section, we will show some cases where the coupled Einstein- $k$ -field equations (6,9) admit a first integral or can be solved exactly. Expressing the energy density of the  $k$ -field as  $\rho = VF/(1 - \gamma)$  and using the conservation equation (8), we get the  $k$ -field equation in terms of the baryotropic index  $\gamma$

$$\left(\frac{\gamma}{\phi}\right)' + 3H\left(\frac{\gamma}{\phi}\right)(1 - \gamma) + \frac{V'}{V}(1 - \gamma) = 0. \quad (31)$$

where  $V' = dV/d\phi$ . This form of writing the field equation allows to show that the first integral of the  $k$ -field equation (9) for any function  $F$  is giving by

$$\frac{\gamma}{\phi} = \phi^{-1} \left( \frac{2}{3H} + \frac{c}{a^3 H^2} \right), \quad (32)$$

where we have assumed an inverse square potential

$$V = \frac{V_0}{\phi^2}, \quad (33)$$

and  $c$  is an arbitrary integration constant. The usual linear field solution  $\phi = \phi_0 t$  along with the power law scale factor  $a = t^{2/3\gamma}$  (with constant  $\gamma$ ) is obtained from the last equation for a vanishing integration constant. Combining Eqs. (5), (7) and (33) with Eq. (32), it can be rewritten as

$$\dot{\phi}F_x - \left[ H + \frac{3c}{2a^3} \right] \frac{\phi}{V_0} = 0, \quad (34)$$

or as

$$-V_0 F_x \dot{H} = \left[ H + \frac{3c}{2a^3} \right]^2. \quad (35)$$

Then, for the tachyonic potential (33), Eqs. (32), (34), (35) are three different forms of writing the first integral of the  $k$ -field equation (9) or (31).

At this point, using Eqs. (5), (33) and (32), it is interesting to see the coupled Einstein- $k$ -field equations (6,9) as a system of differential equations for the function  $F$ . Hence, integrating we obtain

$$F = \frac{3h_0^2}{V_0} + \sqrt{-x} \left[ b + \frac{3c}{2V_0} \int \frac{(2h_0 + ch)h}{(-x)^{3/2}} dx \right], \quad (36)$$

$$H\phi = -\frac{3c}{2} \int \frac{d\phi}{a^3} = h_0 + ch, \quad (37)$$

where  $h_0$  and  $b$  are arbitrary constants and  $\dot{h} = -3\dot{\phi}/2a^3$ . In the particular case  $c = 0$ , the Eq. (36) becomes

$$F^\infty = \frac{3h_0^2}{V_0} + b\sqrt{-x}, \quad (38)$$

which, after a redefinition of the constants, turns into the function  $F_{1/2}$  that generates the ETF,  $T_{1/2}$ , (see Eqs. (13,14) for  $r = 1/2$ ). For this "divergent"  $k$ -essence theory the sound speed (10) diverges and the  $k$ -field equation (9) becomes a first order equation that is only consistent with an inverse square potential. This divergent model is related to the linear  $k$ -field model,  $\phi = \phi_0 t$ , obtained by evaluating  $F$ ,  $F_x$  at  $x = x_0 = -\phi_0^2$  into the Einstein- $k$ -field equations (6,9). The linear  $k$ -field model driven by the potential (33) leads to the power-law solutions  $a = t^\lambda$  with

$$\lambda = \frac{1}{3} \frac{f + 2\phi_0^2 f'}{\phi_0^2 f'}, \quad V_0 = \frac{\lambda}{f'}, \quad (39)$$

where we have defined  $f = F(-\phi_0^2)$ ,  $f' = F_x(-\phi_0^2)$ .

From Eqs. (38,39) it is easy to show that the linear model is isomorphic to the divergent one. This can be done by constructing a one-to-one mapping between these two models. In

fact, choosing in the divergent model the constants  $h_0^2 = \lambda^2 \phi_0^2$  and  $b = -2\lambda\phi_0/V_0$  we find the same power-law solutions obtained from the linear model. In Ref. [28] it was suggested that this might be the reason as to why a model with a diverging sound speed leads to serious problems as discussed in a recent paper [37]. In addition, assuming a series expansion of the function  $F(x)$  around  $x = x_0$ , the background cosmology is completely determined by the first two coefficients  $(f, f')$  of the expansion of  $F$  and the value of  $\phi_0$ . Hence, the model is insensitive to the remaining coefficients in the expansion of the function  $F$  and both the linear and divergent models should be considered as equivalent. This means that the power-law solutions generated by an inverse square potential possesses a degeneracy. Possibly this degeneracy may be removed by perturbing the solutions.

#### A. Inverse square potential and linear function $F$

From the  $k$ -field equation (35), it can be seen that a linear function

$$F = 1 + mx, \tag{40}$$

where  $m$  is a constant, decouples the dynamics of the background geometry from the dynamics of the  $k$ -field. Therefore, this choice clearly introduces a rather small degree of nonlinearity into the dynamical equations allowing us to obtain the general solution of the coupled Einstein- $k$ -field equations. On the other hand, the function (40) mimics the behaviour of other models. For instance, when  $x \ll 1$  the tachyonic function  $F = (1 + x)^{1/2}$  can be approximated by  $F \approx 1 + x/2$  and it has the form (40) [8]. In Ref. [25] it was introduced a set of models where  $F$  admits a power series expansion similar to (40). This form is reminiscent of a Born-Infeld action with higher order corrections in  $x$ , and particular cases were investigated in [26, 37]. Hence, the knowledge of the general solution for the linear function (40) should be of interest, at least for understanding the asymptotic behaviour of many others models generated by an analytical function  $F(x) = F(0) + F_x(0)x + \dots$ . In fact, keeping the first order term in the expansion of  $F$ , it adopts the linear form (40), after redefining the potential  $V$  to set the constant  $F(0) = 1$ .

Combining Eqs. (35) and (40), we obtain the following nonlinear second-order differential equation for the scale factor

$$\frac{d^2 s}{d\tau^2} + s^\sigma \frac{ds}{d\tau} + \frac{1}{4}s^{2\sigma+1} = 0, \quad \sigma = -3mV_0, \quad (41)$$

where we have used the new variables  $s$  and  $\tau$ , defined by

$$s = a^{-3/\sigma}, \quad \tau = \frac{3c}{mV_0}t. \quad (42)$$

The general solution of Eq. (41) can be found changing to the nonlocal variables  $z$  and  $\eta$ , defined by [38]

$$z = \frac{s^{\sigma+1}}{\sigma+1}, \quad \eta = \int s^\sigma d\tau, \quad \sigma \neq -1, \quad (43)$$

$$z = \ln s, \quad \eta = \int \frac{d\tau}{s}, \quad \sigma = -1. \quad (44)$$

Then, in these new variables, the Eq. (41) becomes a linear homogeneous differential equation with constant coefficients

$$\frac{d^2 z}{d\eta^2} + \frac{dz}{d\eta} + \frac{\sigma+1}{4}z = 0, \quad \sigma \neq -1, \quad (45)$$

equivalent to a damped harmonic oscillator equation, and

$$\frac{d^2 z}{d\eta^2} + \frac{dz}{d\eta} + \frac{1}{4}z = 0, \quad \sigma = -1. \quad (46)$$

On the other hand, expressing the Eq. (34) for the  $k$ -field in terms of the independent variable  $\eta$ , we get

$$\frac{d\phi}{\phi d\eta} = -\frac{3da}{\sigma a d\eta} + \frac{1}{2}. \quad (47)$$

Once  $z(\eta)$  is known from Eq. (45), one can compute  $s(\tau)$  from Eq. (43), the scale factor  $a(\tau)$  from (42), and the  $k$ -field  $\phi(\tau)$  from equation (34). Following this procedure and inserting the general solutions of Eqs. (45) and (47) into the Einstein equation (6), we find the scale factor, the  $k$ -field and the relationship amongst the integration constants

$$a = \left[ \sqrt{-B} e^{-\frac{\eta}{2}} \sinh \left( \frac{\sqrt{-\sigma}}{2} \eta + \eta_0 \right) \right]^{-\sigma/3(\sigma+1)}, \quad \sigma < -1, \quad (48)$$

$$a = a_0 e^{-\eta/12 + V_0 e^{-\eta/81c^2\phi_0^2}}, \quad \sigma = -1, \quad (49)$$

$$a = \left[ \sqrt{B} e^{-\frac{\eta}{2}} \cosh \left( \frac{\sqrt{-\sigma}}{2} \eta + \eta_0 \right) \right]^{-\sigma/3(\sigma+1)}, \quad -1 < \sigma < 0, \quad (50)$$

$$a = \left[ \sqrt{B} e^{-\frac{\eta}{2}} \sin \left( \frac{\sqrt{\sigma}}{2} \eta + \eta_0 \right) \right]^{-\sigma/3(\sigma+1)}, \quad 0 < \sigma, \quad (51)$$

the  $k$ -field is given by

$$\phi = \phi_0 a^{-3/\sigma} e^{\eta/2}, \quad (52)$$

where

$$B = \frac{4(\sigma+1)V_0}{27c^2\phi_0^2}, \quad (53)$$

$\eta_0$  and  $\phi_0$  are arbitrary integration constant.

For  $\sigma < -1$ , the solution expands from a singularity as  $t^{1/3}$  and ends as  $t^{-\sigma/3}$ . When  $\sigma > -3$  the scale factor displays a power-law inflationary scenario. For  $-1 < \sigma < 0$ , the universe expands from a singularity as  $t^{1/3}$  and its final behaviour is given by  $t^{1/3}$ . For  $\sigma > 0$  the solution represents a contracting universe which begins at a finite time, reaches a minimum where it bounces, exhibiting a final superaccelerated expansion  $a(t) \propto (t_0 - t)^{-\sigma/3}$ . The universe has a finite time span and bounces when the  $k$ -field satisfies the condition  $\dot{\phi}^2 = -1/m$ .

As this model displays an accelerated expanding stage at late times it may be an interesting alternative to describe the epoch where dark energy dominates.

## B. The explicit solution

For  $\sigma = -4$  or  $mV_0 = 4/3$ , we can solve the Eq. (41) by means of the substitution

$$s^{-4} = \frac{1}{2} \frac{v^{-4}}{\int v^{-4} d\tau}, \quad (54)$$

so that Eq. (41) reads as

$$\ddot{v} = 0. \quad (55)$$

Inserting its solution into (54), using (42) and integrating (47), we get the general solution for the evolution and the  $k$ -field satisfying the Eq. (6):

$$a(\tau) = \left(\frac{3}{2}\right)^{1/3} \left[ \frac{9c^2 V_0}{16\phi_0^2} t^4 - ct \right]^{1/3}, \quad (56)$$

$$\phi(\tau) = \frac{2}{3t^{1/2}} \left[ -\frac{\phi_0^2}{c} + \frac{9V_0}{16} t^3 \right]^{1/2}, \quad (57)$$

where we have chosen  $c < 0$  to set the singularity at  $t = 0$ . The scale factor (56) exhibits a transition from  $a \propto t^{1/3}$  to an accelerated expansion where  $a \propto t^{4/3}$ . Curiously, it coincides with the solutions found in [39] where it was considered a FRW spacetime filled with a scalar field driven by an exponential potential. In the next section we will investigate this relation between the scalar field and the  $k$ -essence field. The  $k$ -field diverges at the singularity as  $\phi \propto t^{-1/2}$  and behaves as  $\phi \propto t$  for large time. It has a minimum and a turning point where the kinetic energy vanishes.

Another set of solutions can be found when  $c = 0$ . Here, the Eq. (35) reduces to  $\dot{H} = -H^2/mV_0$  and we get the power-law expansion  $a = t^{mV_0}$  and a linear  $k$ -field  $\phi = \phi_0 t$ .

### C. The polynomial function $F_\gamma(x) = (-x)^{\frac{\gamma}{2(\gamma-1)}}$

This polynomial function yields a constant baryotropic index  $\gamma$  and the power-law expansion  $a = t^{2/3\gamma}$  [28]. For  $F_\gamma$  the general solution of Eq. (34) is

$$\phi = \left[ b + 4\phi_0 t^{(2-\gamma)/\gamma} \right]^{\gamma/(2-\gamma)}, \quad \gamma \neq 2, \quad (58)$$

$$\phi = \phi_0 t^b, \quad \gamma = 2, \quad (59)$$

where  $b$  and  $\phi_0$  are integration constants. Consistency between these solutions and the Eq. (6) gives the following relation between the integration constants,

$$V_0 = \frac{4(1-\gamma)}{3\gamma^2} (4\phi_0)^{\gamma/(1-\gamma)}, \quad \gamma \neq 2 \quad (60)$$

$$b = \pm \frac{1}{\sqrt{3V_0}}, \quad \gamma = 2. \quad (61)$$

For large cosmological time the  $k$ -field behaves as  $\phi \approx t$ . More details about the cosmological model generated by the set  $F_\gamma$  can be found in Ref. [28].

#### D. Constant potential case

We have seen that the generalized Chaplygin gas model was proposed as unified dark matter. It is derived from a Lagrangian containing non-standard kinetic-energy terms (i.e., non-quadratic) and can be considered as driven by a constant potential. Below we show that, even in absence of any potential energy term, a general class of models exists, including the ECG, connecting a dust dominated era at early times with an accelerated expansion stage at late times.

For a constant potential  $V = V_0$ , the  $k$ -field equation (31) has the first integral

$$\frac{\gamma}{\dot{\phi}} = \frac{c}{a^3 H^2}, \quad (62)$$

or, after using Eqs. (5) and (7), it turns into

$$a^3 F_x \dot{\phi} = \frac{3c}{2V_0}, \quad (63)$$

where  $c$  is an arbitrary integration constant. Also, combining them with the Friedmann equation (6), the baryotropic index associated with this kind of  $k$ -essence can be written in a more convenient form

$$\gamma = \frac{1}{1 + 2V_0^2 a^6 F F_x / 9c^2}. \quad (64)$$

From the last equation we see that for a large set of model, i.e., the class of model generated by the set of functions  $F$  satisfying the condition  $a^6 F F_x \ll 1$  at early time, the universe is dust dominated in the beginning. At intermediate times it behaves as it were filled with a perfect fluid with equation of state  $p \propto \rho$ . Finally, the universe ends in an accelerated expansion scenario. So, these alternative models play the same role that the generalized Chaplygin gas i.e., interpolating between dark matter at early time and dark energy at late time.

Now, we investigate a simple kinetic  $k$ -essence model generated by a function  $F$  satisfying the more general condition  $a^6 F F_x \approx \text{constant}$  at early time. This model is generated by the following function

$$F = \frac{1}{(2\alpha - 1)V_0} \left[ 2\alpha\alpha_0\sqrt{-x} - (-x)^\alpha \right], \quad (65)$$

where  $\alpha$  and  $\alpha_0$  are two real constants. The energy density and pressure of the  $k$ -field are calculated from Eqs. (4) and (65)

$$\rho = (-x)^\alpha, \quad p = -\frac{1}{(2\alpha - 1)} \left[ 2\alpha\alpha_0\sqrt{-x} - (-x)^\alpha \right], \quad (66)$$

the equation of state is

$$p = \frac{1}{2\alpha - 1} \left[ \rho - \frac{2\alpha\alpha_0}{\rho^{-1/2\alpha}} \right], \quad (67)$$

and the sound speed becomes

$$c_s^2 = \frac{1}{2\alpha - 1} \left[ 1 - \frac{\alpha_0}{\rho^{(2\alpha-1)/2\alpha}} \right]. \quad (68)$$

Solving the conservation equation (8), we obtain the energy density

$$\rho = \left[ \alpha_0 + \frac{c_0}{a^3} \right]^{2\alpha/(2\alpha-1)}, \quad (69)$$

where  $c_0$  is a redefinition of the integration constant  $c$ . Restricting to positive sound speed, the source (69) and the ECG induce a similar evolution of the scale factor. However, there is a significant change when both constant  $\alpha_0$ ,  $c_0$  are positive and  $\alpha > 1/2$ , because in this case the baryotropic index becomes

$$\gamma = \frac{2\alpha}{2\alpha - 1} \left[ 1 - \frac{\alpha_0}{\rho^{(2\alpha-1)/2\alpha}} \right]. \quad (70)$$

So, near the singularity, the energy density diverges and the baryotropic index behaves as  $\gamma \approx 2\alpha/(2\alpha - 1)$ , indicating that the model begins to evolve with a power-law dominated phase, where the scale factor is  $a \propto t^{(2\alpha-1)/3\alpha}$ . In this limit the sound speed (68) behaves as  $c_s^2 \approx (2\alpha - 1)^{-1}$ . For large  $\alpha$ , the model is initially dust dominated, with approximated vanishing sound speed, approaching to the ECG generated by the source (28) in the limit of large  $r$ . At late times the model ends in a de Sitter stage. Such "modified Chaplygin gas" may be considered as an alternative model to the generalized Chaplygin gas investigated in [31]. It allows the evolution of the initial perturbations in the energy density into a nonlinear regime to form a gravitational condensate of particles that could play the role of cold dark matter. The cool dark matter condensates gravitationally into the regions where the pressure  $p \approx 0$  and the  $k$ -field is close to  $\phi_c = (2\alpha\alpha_0)^{1/(2\alpha-1)}$ . In this case, the model

yields an energy density which scales like the sum of a non-relativistic dust component at  $\phi = \phi_c$  with equation of state  $p = 0$  and a cosmological-constant-like component  $p = -\rho$ .

#### IV. LINKING SCALAR AND $k$ -ESSENCE COSMOLOGIES

The observed acceleration of the present Universe has been investigated assuming that the dark energy can be described by quintessence and more recently by  $k$ -essence . The last one involves an effective scalar field theory generated by a Lagrangian with a non-canonical kinetic term. Particular cases of  $k$ -essence are generalized Chaplygin gas and tachyon dark energy models. Quintessence and  $k$ -essence frameworks are usually based in a homogeneous scalar fields driven by an exponential potential in the case of quintessence or an inverse square potential in the case of  $k$ -essence. Both encounter the so-called coincidence problem, namely, why are the energy densities of dark energy and dark matter of the same order today. Standard quintessence model appear promising in this point, as it can solve this problem for flat FRW universes provided the dark matter component is assumed to be dissipative [40]. The system is attracted to a stationary and stable solution characterized by the constancy of both density parameters at late times. In addition, a class of  $k$ -essence models has been claimed to solve the coincidence problem by linking the onset of dark energy domination to the epoch of matter domination [24, 25]. From these satisfactory results and taking into account that both models involve evolving scalar fields, we believe it is reasonable to explore whether quintessence and  $k$ -essence frameworks have same kind of similitudes. We will deal with this question by investigating in which cases quintessence resembles  $k$ -essence.

Another interesting aspect to be considered is related with current observations which would indicate that the universe is superaccelerated and it could be filled with a non-standard fluid that violates the weak energy condition. In this sense, it was recently proposed a kind of matter described by an homogeneous scalar field with negative kinetic energy term. The fluid with negative pressure obeys and equation of state of the form  $p = (\gamma - 1)\rho$ , where  $\gamma$  is taken negative and the models are known as phantom or ghost cosmologies [41]-[45]. The phantom and quintessence cosmologies can be investigated simultaneously by taking a scalar field with both signs of the kinetic term driven by an exponential potential. The dynamical equations of these cosmological models are

$$3H^2 = \frac{1}{2}q\dot{\varphi}^2 + \mathcal{V} \quad (71)$$

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{1}{q}\frac{d\mathcal{V}}{d\varphi} = 0, \quad (72)$$

$$\mathcal{V}(\varphi) = \mathcal{V}_0 e^{-qA\varphi}, \quad (73)$$

where  $q$ ,  $A$  are real numbers and  $\mathcal{V}_0$  is a positive constant. The exponential potential is interesting because it may be considered just as a limit of a more complex potential [44]. For negative values of  $q$  the above equations describe a phantom cosmology [45].

It will be demonstrated that the scale factor obtained from Eqs. (71)-(73) or from the  $k$ -essence model generated by the linear function  $F = 1 + mx$ , and driven by an inverse square potential (see Eqs. (6), (33), (35) and (40)) is the same. To this end, we sketch the procedure followed in the Ref. [46]. From Eqs. (71)-(73), we find

$$\dot{H} = -\frac{1}{2}q\dot{\varphi}^2, \quad (74)$$

and the first integral of the Klein-Gordon equation (72)

$$\dot{\varphi} = AH + \frac{c_1}{a^3}, \quad (75)$$

where  $c_1$  is an arbitrary integration constant. Now, inserting Eq. (75) into (74), we obtain the second-order differential equation for the scale factor

$$\frac{d^2 S}{d\zeta^2} + S^\nu \frac{dS}{d\zeta} + \frac{1}{4}S^{2\nu+1} = 0, \quad \nu = -6/qA^2, \quad (76)$$

where we have used the new variables  $S$  and  $\zeta$ , defined by

$$S = a^{-3/\nu}, \quad \zeta = c_1 q A t. \quad (77)$$

With the following identification of the parameters

$$mV_0 = \frac{2}{qA^2}, \quad \frac{3c}{2} = \frac{c_1}{A}, \quad (78)$$

the Eqs. (41) and (76) coincide. Therefore, both models are described by the same scale factor and they are geometrically equivalent. Also, from Eqs. (6,7), (40) and Eqs. (71,72), we get a relationship between both potentials

$$3H^2 + \dot{H} = V(t) = \mathcal{V}(t), \quad (79)$$

showing they are the same function of the cosmological time  $t$ , so that

$$\frac{V_0}{\phi^2} = \mathcal{V}_0 e^{-qA\varphi}. \quad (80)$$

After inserting the  $k$ -field (52) into the last equation we find the scalar field

$$\varphi = \frac{1}{qA} \ln \varphi_0 + \frac{\eta}{qA} + A \ln a, \quad (81)$$

where  $\varphi_0 = \mathcal{V}_0 \phi_0^2 / V_0$ . The Eq. (80) supplies the link between the scalar field and the  $k$ -essence field

$$\phi = \frac{\phi_0}{\varphi_0^{1/2}} e^{qA\varphi/2}, \quad (82)$$

displaying that these models are dynamically no equivalent. Thus, the homogeneous quintessence and phantom fields are different than  $k$ -essence field.

Resuming, from the cinematocal point of view, that is the background geometry, we have obtained exact equivalences and it is impossible to differentiate between quintessence,  $k$ -essence and phantom cosmologies because they share the same scale factor. To distinguish between them, it appears necessary to focus our attention on the scalar field.

## V. CONCLUSIONS

We have investigated the set of  $T_r$ -fields whose effective sound speed is proportional to the sound speed of the tachyon field. They can be grouped into three types according whether they yield phantom expansion or accelerated expansion with or without inflation. We have shown these behaviours by finding exact power-law solutions for an inverse square potential and proved that the  $T_{1/2}$ -field is compatible only with this potential. Each  $T_r$ -field produces an ECG and the set of all these gases can be divided into three kinds, one of which contains the generalized Chaplygin gas and the others give rise to "perfect fluids", leading to new

evolutions. There exists basically nonsingular bouncing solutions which interpolate between two superaccelerated stages or singular ones with a finite time span, as well as, peculiar singular solutions that begin with a finite scale factor. In this manner, both, the ETF and the ECG may be considered fair candidates to implement phantom cosmologies.

For an inverse square potential, we have found the first integral of the  $k$ -field equation for any function  $F$  and shown that the coupled Einstein- $k$ -field equations can be solved in some cases. In particular, the divergent  $k$ -essence theory generated by the  $T_{1/2}$ -field becomes an intrinsic component of all  $k$ -essence models. Therefore, for power-law expansions the linear  $k$ -field model driven by an inverse square potential and the divergent model are isomorphic. We have obtained the general solution of Einstein- $k$ -field equations for a linear function  $F$ . From the kinematical point of view this model and the quintessence scalar field one driven by an exponential potential are the same. However, they are dynamically nonequivalent, because the  $k$ -field and the scalar field are linked by the Einstein equation, e.g., both potentials are the same function of the cosmological time.

For a constant potential, we have studied a  $k$ -essence field associated with a perfect fluid whose equation of state contains a term proportional to the energy density and the other has the form of an ECG. This model, essentially different than the ECG model, smoothly interpolates between a power-law dominated phase and a de Sitter phase. In this "modified Chaplygin gas" scenario, it may be chosen the value of the  $k$ -field where initial perturbations condensate gravitationally in cold dark matter.

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